Journal of Education and Teaching (JET)
Vol. 7 No. 3 2026

Polya–Van Hiele Integration Hierarchical Model in Students' Geometric Thinking Processes

Venissa Dian Mawarsari[1], Martyana Prihaswati[2], Eko Andy Purnomo[3], YL Sukestiyarno[4], Scolastika Mariani[5], Iwan Junaedi[6]
Mathematics Education, Universitas Muhammadiyah Semarang, Indonesia[1,2,3]
Mathematics Education, Universitas Negeri Semarang, Indonesia [4,5,6]
Corresponding email: venissa@unimus.ac.id

Draft article history
Submitted: 03-23-2026;
Revised: 06-18-2026;
Accepted: 06-25-2026;

ABSTRACT: Much research has been done on geometric thinking in junior high school students, but there is still little research that integrates Polya's problem-solving model with Van Tiele's geometric thought stages in a single hierarchical framework. This research aims to develop the Polya–Van Hiele Integrated Hierarchical Model to describe students' geometric thinking processes in solving geometry problems. This study uses a qualitative approach with a case study design. The research subjects were selected based on the level of geometric thinking ability in the students. Data was collected through geometric thinking tests and in-depth interviews, then analyzed using qualitative descriptive methods. The results of the study show that students' geometric thinking processes develop progressively and are in harmony with the stages of Polya problem-solving, namely understanding problems, making plans, executing plans, and looking back. At the planning stage, students demonstrate skills at the level of visualization and analysis. Furthermore, at the stage of planning implementation, geometric thinking develops towards the level of informal deduction, formal deduction, and precision. At the level of rigor, students are able to provide valid mathematical justifications, compare, and generalize geometric concepts. This research resulted in the Polya-Van Hiele Integrated Hierarchical Model, which shows that the development of geometric thinking is embedded in the process of solving mathematical problems. This model theoretically integrates Polya's problem-solving theory and Van Hiele's theory of geometric thinking in the hierarchical structure of students' cognitive development, as well as practically helping teachers design geometry learning activities that support students' thinking development at each geometric level.

Keywords: geometric thinking process, hierarchical model, plane geometry, polya problem-solving, van hiele theory.

ABSTRAK: *Banyak penelitian telah dilakukan mengenai kemampuan berpikir geometris pada siswa sekolah menengah pertama, namun masih sedikit penelitian yang mengintegrasikan model pemecahan masalah Polya dengan tahapan pemikiran geometris Van Hiele dalam satu kerangka kerja hierarkis. Penelitian ini bertujuan untuk mengembangkan Model Hierarkis Terintegrasi Polya–Van Hiele guna mendeskripsikan proses pemikiran geometris siswa dalam memecahkan masalah geometri. Studi ini menggunakan pendekatan kualitatif dengan desain studi kasus. Subjek penelitian dipilih berdasarkan tingkat kemampuan pemikiran geometris siswa. Data dikumpulkan melalui tes pemikiran geometris dan wawancara mendalam, kemudian dianalisis menggunakan metode deskriptif kualitatif. Hasil penelitian menunjukkan bahwa proses pemikiran geometris siswa berkembang secara progresif dan selaras dengan tahapan pemecahan masalah Polya, yaitu memahami masalah, menyusun rencana, melaksanakan rencana, dan memeriksa kembali. Pada tahap perencanaan, siswa menunjukkan* *keterampilan pada tingkat visualisasi dan analisis. Selanjutnya, pada tahap pelaksanaan rencana, pemikiran geometris berkembang menuju tingkat deduksi informal, deduksi formal, dan presisi. Pada tingkat rigor, siswa mampu memberikan justifikasi matematis yang valid, serta membandingkan dan menggeneralisasi konsep-konsep geometri. Penelitian ini menghasilkan Model Hierarkis Terintegrasi Polya-Van Hiele, yang menunjukkan bahwa perkembangan pemikiran geometris terintegrasi dalam proses pemecahan masalah matematika. Secara teoretis, model ini memadukan teori pemecahan masalah Polya dan teori pemikiran geometris Van Hiele dalam struktur hierarkis perkembangan kognitif siswa, serta secara praktis membantu guru merancang kegiatan pembelajaran geometri yang mendukung perkembangan pemikiran siswa pada setiap tingkat geometris.*

Kata kunci*: proses berpikir geometris, model hierarkis, geometri bidang, pemecahan masalah Polya, teori Van Hiele.*

INTRODUCTION

Geometric thinking is an essential component of mathematics learning, particularly in junior high school education. (Amelia & Anugrahana, 2024; Rizkayati & Sari, 2026). Geometry learning not only develops students' spatial visualization abilities but also enhances logical, systematic, and abstract reasoning skills that are important in everyday problem-solving (Ramadhan et al., 2021; Naja et al., 2021; Rahayu et al., 2023). However, many studies have reported that students still experience difficulties in understanding geometric concepts, including identifying geometric properties, relating concepts, and constructing deductive arguments (Baeti & Murtalib, 2019; Rahmah, 2021; Umami & Asdarina, 2024). These difficulties indicate that students' geometric thinking development has not been fully optimized in geometry instruction.

One important framework for understanding students' geometric thinking is Van Hiele's theory, which classifies geometric thinking into five hierarchical levels: visualization, analysis, informal deduction, formal deduction, and rigor (Mawarsari, Rochmad, et al., 2024). Each level represents a distinct mode of reasoning, and students progress through these levels gradually through appropriate learning experiences (Mawarsari, Sukestiyarno, & Prihaswati, 2024). Previous studies have shown that students' success in solving geometry problems is closely related to their level of geometric thinking (Rejeki et al., 2021; Celik & Yilmaz, 2022; Mawarsari, Kintoko, et al., 2023). Most junior high school students are generally still at the visualization and analysis levels (Mawarsari, Waluya, et al., 2023), which often limits their ability to solve problems requiring higher-order deductive reasoning.

Most Junior High School students are expected to still be at the level of visualization and analysis (Mawarsari, Waluya, et al., 2023). Research shows that students' understanding of geometry depends not only on the knowledge they have, but also on their cognitive Development stage in geometric thinking (Cesaria et al., 2021; Siregar & Elvi, 2024). An immature understanding of flat and spatial shapes often leads to errors in classifying shapes, deducing properties, or solving problems that require more complex reasoning (Rong & Mononen, 2022). Students also have difficulty in identifying the properties of buildings, connecting geometric concepts, and formulating logical arguments in solving geometric problems (Ngirishi & Bansilal, 2019; Utomo et al., 2023). These difficulties show

Corresponding author: Venissa Dian Mawarsari (venissa@unimus.ac.id)

that geometry instruction has not fully developed students' geometric thinking processes at their level of cognitive Development.

Students are often asked to solve geometry problems that require a high level of reasoning, while they are still at a lower level of thinking. As a result, students have difficulty understanding geometric concepts and solving problems related to them (Triyani et al., 2024). In fact, understanding students' cognitive processes is very important for understanding how students approach problems, represent geometric situations, analyze the properties of buildings, use relationships between concepts, and reason when solving problems (Aktaş & Ünlü, 2017; Arnal & Víctor, 2024). This problem highlights the importance of examining students' thinking processes when solving geometry problems, particularly at different levels of geometric thinking. Basically, when students face a math problem, they carry out a problem-solving process (Toker, 2021; Rizkayati & Sari, 2026). This process involves various thinking activities such as understanding the problem, planning a solution strategy, implementing a plan, and re-evaluating the solutions obtained.

In solving geometry problems, students also engage in a mathematical problem-solving process. One of the most widely used frameworks in mathematics education is Polya’s problem-solving model, which consists of four stages: understanding the problem, devising a plan, carrying out the plan, and looking back (Purnomo et al., 2022; Purnomo et al., 2024). This model provides a systematic structure that guides students in solving mathematical problems logically and sequentially (Budayasa & Rahaju, 2025). Nevertheless, the effectiveness of problem-solving processes in geometry is strongly influenced by students’ levels of geometric thinking (Uygun, 2022). Students at different Van Hiele levels tend to approach and interpret geometry problems differently (Wulandari et al., 2021).

Therefore, the problem-solving process needs to be understood by considering the level of Development of students' geometric thinking (Kholid & Aisyah, 2025). Although many studies have examined students’ geometric thinking based on Van Hiele’s theory and others have investigated mathematical problem-solving using Polya’s framework, these two perspectives are generally studied separately. There remains limited research integrating Polya’s problem-solving stages with Van Hiele’s levels of geometric thinking to explain how students’ geometric thinking develops during geometry problem-solving activities (Naufal, 2021; Mahlaba & Mudaly, 2022; Kandaga et al., 2022; Celik & Yilmaz, 2022; Trimurtini et al., 2023; Mawarsari, Sukestiyarno, Mariani, et al., 2024). Previous studies have highlighted students’ difficulties in solving geometry problems, yet they rarely connect these difficulties with the cognitive development stages proposed by Van Hiele within Polya’s problem-solving framework.

Based on these problems, this study proposes an integrated framework that combines Polya’s problem-solving stages with Van Hiele’s geometric thinking levels to provide a more comprehensive understanding of students’ geometric thinking processes in geometry problem-solving. This study aims to develop the Polya–Van Hiele Integrated Hierarchical Model to analyze the relationship

Corresponding author: Venissa Dian Mawarsari (venissa@unimus.ac.id)

between students' problem-solving processes and their geometric thinking levels. The proposed model is expected to provide a systematic description of how geometric thinking develops across the stages of mathematical problem-solving and to contribute both theoretically and practically to geometry learning and instruction.

RESEARCH METHOD

Research Design

This study uses an exploratory qualitative research design to examine in depth the geometric thinking process of students in solving geometry problems, based on the Van Hiele geometric thinking levels and the Polya problem-solving stages. The exploratory design was chosen because this research not only aims to describe students' abilities but also to explore in detail how the geometric thinking process develops at each stage of problem-solving. Through this approach, researchers can understand the relationship between students' geometric thinking levels and the strategies they use in understand problems, plan solutions, implement plans, and re-evaluate solutions, according to the stages of the Polya problem-solving model. The results of the exploration were then used to formulate the Polya-Van Hiele Integrated Hierarchical Model in the geometric thinking process of junior high school students.

Research Subject

The subject of the study is a grade VII junior high school student studying field geometry material. The selection of grade VII students is based on the consideration that at this level, students begin to learn basic concepts of geometry more formally and show variations in the level of geometric thinking that are in accordance with Van Hiele's theory. In addition, grade VII students are in the transition stage from visual to analytical and deductive thinking, so it is considered appropriate to explore the development of geometric thinking processes in problem solving. The selection of research subjects was carried out using purposive sampling, a deliberate selection of subjects based on the research objectives (Creswell, 2014; Sukestiyarno, 2020). The selection of the research subject was based on the results of the geometric thinking ability test, which was used to identify students' levels of geometric thinking in accordance with Van Hiele's theory. Based on the results of the test, one student was selected who represented each level of geometric thinking ability that appeared in junior high school students, namely: (1) visualization level, (2) analysis level, (3) informal deduction level, (4) formal deduction level, and (5) rigor level. The selected subjects were then analyzed in depth to examine their thinking process in solving geometric problems.

Research Instruments

The instruments used in this study include: (1) geometric thinking ability tests used to identify students' geometric thinking levels based on Van Hiele's theory, (2) semi-structured interview guidelines used to delve deeper into

Corresponding author: Venissa Dian Mawarsari (venissa@unimus.ac.id)

students' thinking processes when solving geometry problems, and (3) documentation in the form of student answer sheets and interview results notes used as research supporting data. The following is a grid of instruments used in the research.

Table 1. Student Geometric Thinking Ability Test Grid

Geometric Thinking Level	Geometric Thinking Level Indicator	Question Indicator	Question Item
Level 0: Visualisasi	Draw and name plane geometric shapes based on the identification of the information provided.	Draw the trajectory and mention the shape of the trajectory in the form of a right triangle formed from the ship's route.	(a), (b), dan (c)
Level 1: Analysis	Mention the properties of geometric shapes.	Mention the properties of a right triangle.	(d)
Level 2: Informal Deduction	The relationship between concepts and geometric shapes.	Calculating the hypotenuse of a right triangle using the Pythagoras theorem to calculate the circumference of a right triangle.	(e)
Level 3: Formal Deduction	Construct mathematical arguments logically.	Calculating the overall distance of the ship's trajectory (Calculating the circumference of a right triangle).	(f)
Level 4: Rigor	Proving the truth of a statement.	Proving distance comparison.	(g)

Table 1 shows a grid of geometric thinking tests at each Van Hiele level. The geometric material used is a flat build that focuses on right triangles. The results of the geometric thinking ability test are used to group students by geometric thinking level according to Van Hiele's theory. The test is structured according to the indicators of each level of geometric thinking, namely visualization, analysis, informal deduction, formal deduction, and rigor, as shown in Table 1. Each question item is designed to represent the characteristics of thinking at a certain level. The level determination process is carried out by analyzing students' answers in terms of accuracy, the way students represent geometric concepts, the use of building properties, the ability to connect concepts, the preparation of logical arguments, and the ability to provide mathematical proof.

Students are categorized at a given level if they consistently meet the main indicators for that level. For example, students at the visualization level can recognize and draw geometric shapes from visual displays, while students at the analysis level can explain the properties of geometric shapes. Students at the informal deduction level can connect concepts and use interstructural relationships in problem-solving, while students at the formal deduction level can systematically formulate logical arguments. Students at the rigor level can provide formal mathematical proofs and justifications. Based on the classification results,

Corresponding author: Venissa Dian Mawarsari (venissa@unimus.ac.id)

one student was selected who best represented each level of geometric thinking to be analyzed more deeply in the process of solving geometry problems.

Instrument Validity Test

The geometric thinking ability test instrument was validated by 5 experts who measured the content and constructs. The following are the results of the validation of the geometric thinking ability test instrument's content.

Tabel 2. Validation of the Content of Diagnostic Instruments for Geometric Thinking Ability

No	Aspects Assessed	V1	V2	V3	V4	V5	Aiken's V	Interpretation
1	Compatibility of the material with the junior high school curriculum.	4	3	4	3	4	0,87	Highly Valid
2	Conformity of the question with the indicator of geometric thinking ability.	4	3	3	4	4	0,87	Highly Valid
3	Conformity of the question to the purpose of the diagnostic.	3	3	4	3	4	0,80	Highly Valid
4	Clarity of the context of the issue.	3	4	3	3	4	0,80	Highly Valid
5	Image illustration accuracy.	4	3	3	4	4	0,87	Highly Valid
6	Understanding of questions by students.	3	3	4	3	4	0,80	Highly Valid

Based on the results in Table 2, the content validity for each aspect ranges from 0.80 to 0.87, with an average of 0.83. This value shows that all aspects assessed are in the category of being very valid, so that the geometric thinking ability test instrument is stated to have a high level of conformity with the measured geometric thinking ability indicator. Furthermore, the results of the construct validation by 5 experts are shown in the following table.

Tabel 3. Validation of Diagnostic Constructs of Geometric Thinking Ability

No	Aspects Assessed	V1	V2	V3	V4	V5	Aiken's V	Interpretation
1	Compatibility of the question with Van Hiele's theory.	4	3	4	3	4	0,87	Highly Valid
2	The ability of questions to uncover geometric thinking processes.	3	4	3	3	4	0,80	Highly Valid
3	The relationship of geometry concepts in the problem.	4	3	4	4	4	0,87	Highly Valid

Corresponding author: Venissa Dian Mawarsari (venissa@unimus.ac.id)

4	Suitability of the level of difficulty with junior high school students.	3	3	4	3	4	0,80	Highly Valid
5	Ability to identify students' misconceptions.	3	3	4	3	4	0,80	Highly Valid

Based on the results of Aiken's V calculation in Table 3.4, the value of construct validity in each aspect ranges from 0.80 to 0.87, with an average value of 0.83. This value shows that all aspects of the assessment are in the category of being very valid. This shows that the developed instrument in consistent with the geometric thinking ability construct, as defined by Van Hiele's theory. The next instrument is an interview, with the following framework.

Table 4. Geometric Thinking Process Interview Grid

Geometric Thinking Process	Indicators
Understand the Problem	Identifying information in geometry problems.
	Explain the questions in the question.
Devise a Plan	Determine a solution strategy.
	Explain the reasons for choosing a particular strategy.
	Level 0: Visualization 1) How to draw geometric shapes. 2) Visually recognize geometric shapes and shapes.
	Level 1: Analisis 1) Explaining the properties of geometric shapes. 2) Using the wake state to solve problems.
Carry out the Plan	**Level 2: Informal Deduction** Connecting several other concepts in problem-solving.
	Level 3: Formal Deduction Constructing mathematical arguments logically in solving geometric problems.
	Level 4: Rigor Using logical reasoning or simple proof.
Looking Back	Recheck the completion results.
	Checking the correctness of the solution.

Table 4 is an interview grid used in making interview guidelines. Five experts in mathematics education first validated the interview guidelines used in this study to assess the instrument's suitability for the research purpose of exploring students' geometric thinking processes, based on Van Hiele's theory and Polya's problem-solving stages. Validation is carried out by assessing several aspects, namely the suitability of the indicators with the research objectives, language clarity, question comprehension, and the ability of questions to explore students' thinking processes. Based on validation by five experts, the interview guidelines received an average score of 3.68, placing them in the very valid category. This indicates that the interview guidelines meet the eligibility criteria for use in the study. However, some validators have suggested simplifying the language of the questions to make them easier for students to understand. The improvement was

Corresponding author: Venissa Dian Mawarsari (venissa@unimus.ac.id)

then made before the interview guidelines were used in the research data collection process.

The research instruments used in this study consisted of a geometric thinking ability test and an in-depth interview. The geometric thinking ability test is structured around the Van Hiele geometric thinking level indicator. It is designed to reveal the stages of Polya's problem-solving in the process as it unfolds. Each question item not only measures the student's geometric abilities but also explores how the student understands the problem, strategizes, implements a solution, and reevaluates the answer. Thus, the test instrument allows the researcher to identify the relationship between the Van Hiele geometric thinking level and the Polya problem-solving stage in an integrated manner. In-depth interviews are conducted to strengthen the results of the written test by exploring students' reasons, strategies, and thinking processes at each stage of problem-solving and at each level of geometric thinking that arises.

Data Analysis Techniques

Data analysis is carried out through several stages, namely data reduction, data presentation, and conclusion (Miles et al., 2014). At the data reduction stage, test answers and interview results were classified based on the Polya problem-solving stage indicator and the Van Helée geometric thinking level. Furthermore, the data were analyzed to identify the pattern of the relationship between the problem-solving process and students' development of geometric thinking. At the data presentation stage, the results of the analysis were compiled in the form of a description of students' thinking processes at each stage of Polya, which was associated with the Van Hiele geometric thinking level. The integration of the two models is explored by tracing how each level of geometric thinking arises and develops during the student's problem-solving stage. The final results of the analysis were used to formulate the Polya-Van Hele Integrated Hierarchical Model.

This study also addresses the ethical aspects of research. Before data collection began, the researcher obtained permission from the school and explained the research objectives to the participants. All research subjects participated voluntarily, and their identities were kept confidential using codes or initials during data analysis and reporting. Although this study provides an in-depth overview of the integration of Polya's problem-solving stages and Van Hiele's geometric thinking levels, it has some limitations. The number of subjects is limited to grade VII junior high school students, so the research results cannot be generalized widely. In addition, the focus of exploratory qualitative research is more on the depth of analysis of students' thinking processes than on quantitative measurement of students' abilities. Therefore, further research is needed to test models developed in more diverse contexts, educational levels, and geometric materials.

RESULT AND DISCUSSION

Overview of Students' Geometric Thinking Levels

Corresponding author: Venissa Dian Mawarsari (venissa@unimus.ac.id)

The test results revealed that students' geometric thinking abilities remained relatively low. Most students did not reach any Van Hiele level, while the second largest group was at the visualization level. Only a small proportion of students reached the higher levels of analysis, informal deduction, formal deduction, and rigor. Figure 1 illustrates the distribution of students' geometric thinking levels.

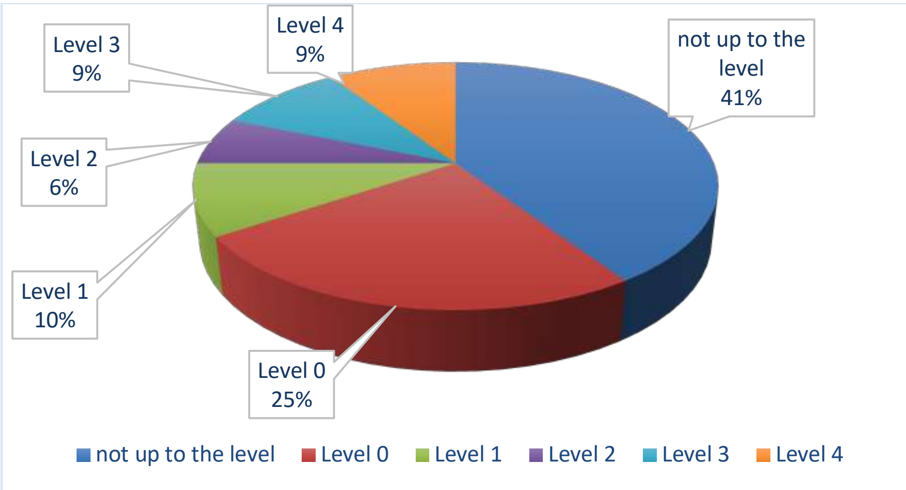


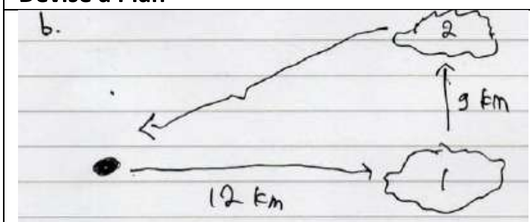
Figure 1. Early Diagnostic Analysis of Geometric Thinking Ability

Based on Figure 1, it is indicated that most students still experience difficulties in developing higher-order geometric reasoning. Students at lower levels tended to rely on visual recognition rather than conceptual understanding. Meanwhile, students at higher levels demonstrated stronger logical reasoning and were more capable of systematically connecting geometric concepts.

Students' Geometric Thinking Based on Van Hiele Levels

Analysis of Students' Geometric Abilities Level 0: Visual

Table 3. Analysis of Students' Geometric Abilities Level 0: Visualization

Answers	Analysis of Geometric Thinking Processes
Understanding the Problem	
a. Kapal Bimasakti yang ada di Pelabuhan akan berlayar ke 2 pulau kecil. Perjalan pertama dilakukan ke Pulau 1, yang ditempuh dengan jarak 12 Km ke arah timur dan dilanjutkan ke arah utara menuju Pulau 2 dengan jarak 3. Setelah dari Pulau 2 langsung kembali ke Pelabuhan	Identify information by writing a statement that is known in the question, but still write it the same as the question.
Devise a Plan	
	**Visualization:** represents the information in the question in the form of a sketch of the ship's sailing route and provides information on the image.
c. Segitiga	Mentioning the shape of the trajectory, but it is not precise, namely, only mentioning a triangle. It should be a right triangle.
d. • ~~[illegible]~~ ~~Segitiga~~ Memiliki 3 sisi • Memiliki 3 sudut	**Analysis:** It is not possible to correctly attribute the properties of a right triangle,

Corresponding author: Venissa Dian Mawarsari (venissa@unimus.ac.id)

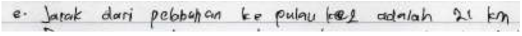
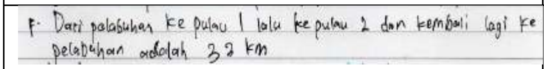
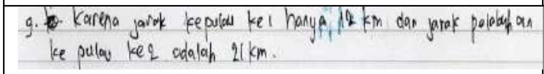
	i.e., the lack of a property of one of the right angles (90°) used to design the solution. So there is an error in determining the distance between island 2 and the port.
Carry out the Plan	
 e. Jarak dari pelabuhan ke pulau ke2 adalah 21 km	**Informal deduction:** constructing students' thinking in planning the solution, namely by determining the distance between island 2 and the port first. However, students were incorrect in determining the distance from the port to Island 2 and did not record the process used to obtain it.
 f. Dari palabuhan ke pulau 1 lalu ke pulau 2 dan kembali lagi ke pelabuhan adalah 32 km	**Formal deduction:** Carry out logical reasoning in solving problems systematically, but incorrectly determine the total distance of the ship's journey from port to Island 1, to Island 2, and back to port. It is not written the process of getting the total distance
 g. Karena jarak ke pulau ke1 hanya 12 km dan jarak pelabuhan ke pulau ke2 adalah 21 km.	**Rigor:** carry out logical reasoning by trying to prove the problem, but the proof provided is incorrect.

Table 3 shows the results of the geometric thinking ability test for Subject 0 (S0). Furthermore, in-depth interviews were conducted, so that the results of both showed that S0's geometric thinking ability was still at the level of visualization (Level 0) according to Van Hiele's theory. However, in the solution, S0 carried out the process of solving the problem from the stage of understanding the problem. In this process, S0 is able to identify important information by writing down the known data on the problem. Furthermore, at the devise a plan stage, S0 represents the problem situation by drawing a sketch of the ship's trajectory and recognizing the geometric shapes formed as right-right triangles based on the visual appearance of the drawing. S0 is also able to mention the basic properties of a right triangle as a basis for planning a solution. However, the results of the interview show that this understanding is still visual and intuitive, because S0 only recognizes the shape based on the visible shape without understanding the geometric relationship between the sides or the concept of angle analytically.

At the carry out the plan stage, S0's geometric thinking ability begins to show the characteristics of informal deduction, which is trying to determine the trajectory of the voyage as the oblique side of a right triangle. However, S0 does not use the Pythagorean Theorem and instead solves the problem by arithmetically adding the length of the sides based on visual logic. At the deduction and rigor stage, S0 has written down the completion steps in a row according to his way of thinking, but the reason given is not appropriate because it is still based on visual perception, not formal geometric relationships. Meanwhile, at the looking back stage, S0 did not re-examine the answers or evaluate the completion process carried out. The results of the interview show that S0 is only aware of the possibility of errors without being able to fix or re-explain the solution systematically. These findings show that the geometric thinking process of S0 is

Corresponding author: Venissa Dian Mawarsari (venissa@unimus.ac.id)

Journal of Education and Teaching (JET) Volume 7 No. 3, 2026
DOI: 10.51454/jet.v7i3.944

still dominated by visual recognition and simple intuition, so it has not progressed towards formal geometric analysis, deduction, and reflection skills.

Level 1 Students' Geometric Ability Analysis: Analysis

Table 4. Analysis of Students' Geometric Abilities Level 1: Analysis

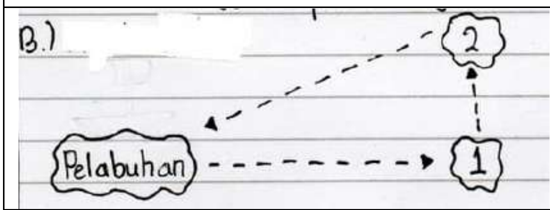
Answers	Analysis of Geometric Thinking Processes
Understanding the Problem	
A.) Diketahui : Jarak pulau 1 : 12 km Jarak pulau 2 : 9 km	Identify information by writing information on the question briefly.
Devise a Plan	
	**Visualization:** represents an illustration of the problem by drawing a sketch of the ship's shipping route according to the information, but not clarified with information on the ship's sailing distance.
C.) Segitiga siku - siku (◿)	Mention the shape of the flat geometry (trajectory of the ship) correctly, i.e. a right triangle based on the identification of the image.
D.) Sudut sikunya 90°, memiliki 3 sisi, memiliki satu sisi miring	**Analysis:** can correctly mention the properties of right-angled triangles, so it is used to plan the completion of the circumference of the triangle.
Carry out the Plan	
E.) Jarak pelabuhan ke pulau 1 ditambah jarak pulau ke 1 ke pulau 2 (12+9) = 21 km.	**Informal deduction:** constructing students' thinking in planning problems by determining the length of the trajectory from port to island 2 as the oblique side of the right triangle. Because it does not use the pyhtagoras formula, the solution is incorrect.
F.) Pelabuhan ke pulau 1 : 12 km, pulau ke 1 ke pulau 2 : 9 km, pulau 2 ke pelabuhan : 21 km, total km : 42 km	**Formal deduction:** have written down the method of completion in detail according to the logic of the student's thinking, but there are errors in the previous completion, so that the results of the trajectory distance are also wrong at any time
G.) Jarak dari pelabuhan ke pulau 2 lebih panjang karena jarak tempuh yang dihitung dari jarak pelabuhan ke pulau 1 ditambah jarak pulau 1 dan pulau 2.	**Rigor:** did not give an answer to the statement, but was immediately given a reason, and the reason given was incorrect, because the student thought that the distance of the port to island 2 was calculated by adding the distance of the port to island 1 and the distance of island 1 to island 2.

Table 4 shows the results of the S1 (S1) geometric thinking ability test. Furthermore, in-depth interviews were conducted, so that the results of the two showed that the geometric thinking skills of S1 were still dominated at the level of

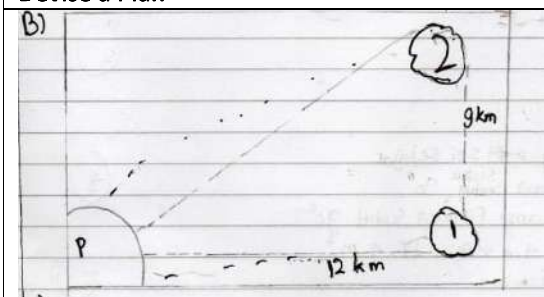
Corresponding author: Venissa Dian Mawarsari (venissa@unimus.ac.id)

visualization and began to develop at the level of analysis according to Van Hiele's theory. However, in solving the problem, S1 carries out the solution process from the stage of understanding the problem. S1 is able to read the problem systematically, identify known and asked information, and rewrite important information as the basis for solving the problem. Furthermore, at the devise a plan stage, S1 is able to represent the ship's trajectory through a more complete sketch drawing accompanied by information about the relationship between points so as to form a right triangle. S1 is also able to recognize the properties of a right triangle, such as having a 90° angle and an oblique side, and begins to relate the concept of the Pythagorean Theorem in developing a solution strategy. These findings show that S1's geometric thinking ability has evolved from mere visual recognition to the ability to analyze the properties of geometric shapes.

In carrying out the plan stage, S1 shows the ability of informal deduction by using the Pythagorean formula to determine the length of the oblique side of a right triangle, and the calculation process is relatively correct. However, at the formal deduction stage, errors are still found in the final result because the simplification process and the use of the previous results have not been carried out appropriately. At the rigor stage, S1 has been able to give reasons for the answers obtained, although the arguments given still depend on the results of imprecise calculations and have not shown formal mathematical generalizations. Meanwhile, at the looking back stage, S1 did not re-examine the answers because he felt that he had solved the questions according to his ability. However, the results of the interview show that S1 began to realize the possibility of a mistake in determining the oblique side of the triangle. This shows that S1's reflective ability is beginning to emerge, although it has not yet developed optimally in improving and evaluating the problem-solving process systematically.

Level 2 Student Geometric Ability Analysis: Informal Deduction

Table 5. Level 2 Student Geometric Ability Analysis: Informal Deduction

Answer	**Analysis Geometric Thinking Process**
Understanding the Problem	
a) diketahui : Jarak ~~Pe~~ dari Pelabuhan ke Pulau = 12km ke arah timur Diketahui : Jarak ke Pulau 1 ke Pulau 2 = 9 km Setelah dari Pulau 2 langsung kembali ke Pelabuhan	Identify the known information in the question clearly.
Devise a Plan	
	**Visualization:** draw a sketch of the ship's sailing route according to the information and provide captions on the image.
C) Segitiga Siku-siku	Mention the shape of the ship's trajectory correctly, namely a right triangle, based on the image.

Corresponding author: Venissa Dian Mawarsari (venissa@unimus.ac.id)

D) Sifat² bangun datar: - Memiliki 3 sisi - Satu sudut 90° - Dua sisi saling tegak lurus - Panjang sisi bisa dihitung	**Analysis:** can correctly mention the properties of right triangles that are used to construct students' thinking in planning problems using the Pythagorean formula.
Carry out the Plan	
E) Jarak dari pelabuhan ke pulau 2 $= a^2+b^2$ $= 12^2+9^2$ $= 144+81$ $= \sqrt{225}$ Jadi Jara kepulau 2 adalah $\sqrt{225}$	**Informal deduction:** performing the calculation of the pyhtagoras formula in determining the length of the oblique sides of a right triangle. The calculation is also correct, only at the end of the settlement the value of $\sqrt{225}$ has not been simplified.
F) Jarak total = 12+9+15 = 35 km	**Formal deduction:** You have written down the way to solve correctly, but the result is wrong.
G) Iya karena Jarak Pelabuhan kepulau 2 15 km, sedangkan pelabuhan ke pulau 1, 12 km	**Rigor:** provide an answer to the statement, accompanied by a reason. However, the existence of previous mistakes resulted in inappropriate reasons.

Table 5 shows the results of the geometric thinking ability test for Subject 2 (S2). Furthermore, in-depth interviews were conducted, and the results of both showed that S2's geometric thinking ability is fairly good and has reached the level of informal deduction according to Van Hiele's theory. At the stage of understanding the problem, S2 can read it thoroughly, identify the known and requested information, and write down the important information as the basis for solving the problem. At the devise-a-plan stage, S2 can represent the ship's trajectory with sketches and recognize the shape formed as a right triangle. In addition, S2 can mention the properties of a right triangle, such as the existence of a 90° angle, two sides perpendicular to each other, and oblique sides, so that it can plan a solution strategy using the Pythagorean Theorem. This shows that S2's geometric thinking skills are no longer based solely on visual recognition but have evolved to include the ability to analyze the properties and relationships among geometric elements.

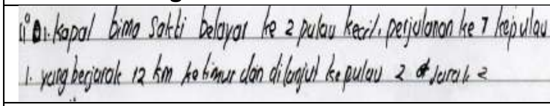
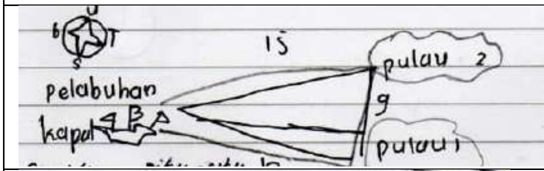
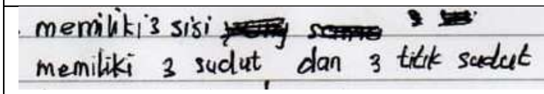
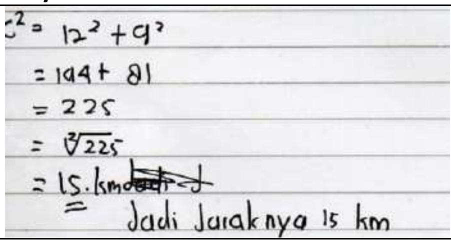
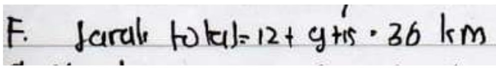
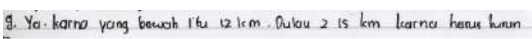
In carrying out the plan stage, S2 demonstrates the ability to deduce informally, using the Pythagorean formula precisely to determine the length of the oblique side of a right triangle. Still, there is an error in the calculation, so it cannot continue with the formal deduction ability. At the rigor stage, S2 writes down the reason, but the reason given is still not completely mathematically correct. Meanwhile, at the looking-back stage, S2 has not independently re-checked the answers, even though they are aware of the importance of this step and can identify possible errors after re-checking. These findings show that S2's conceptual and geometric reasoning abilities have developed well, but its computational accuracy and its ability in reflectively evaluate solutions still need improvement.

Corresponding author: Venissa Dian Mawarsari (venissa@unimus.ac.id)

e-issn: 2746-1467 *Journal of Education and Teaching (JET)* Volume 7 No. 3, 2026
p-issn: 2747-2868 DOI: 10.51454/jet.v7i3.944

Level 3 Student Geometric Ability Analysis: Formal Deduction

Table 6. Level 3 Student Geometric Ability Analysis: Formal Deduction

Answer	Analysis of Geometric Thinking Processes
Understanding the Problem	
	Write down the information that is known in the question clearly.
Devise a Plan	
	**Visualization:** draw a sketch of the ship's sailing route according to the information and provide captions on the image. It's just that there are 2 (two) lines from the port to Island 1.
	Mention the shape of the flat geometry (the trajectory of the ship) correctly, namely a right triangle.
	**Analysis:** mentions the properties of right triangles, but no specific properties of right triangles are shown. It's just that students can plan solutions using the Pythagoras formula.
Carry out the Plan	
	**Informal deduction:** The formula used is correct, namely the Pythagorean theorem. The calculation is also correct.
	**Formal deduction:** performing logical reasoning and calculations for the entire distance of the ship's trajectory using the formula for the circumference of a right triangle.
	**Rigor:** provide answers to statements and accompany them with reasons. However, the reasons given are not precise.

The results of the geometric thinking ability test in Table 6 are an analysis of Subject 3 (S3). Furthermore, in-depth interviews revealed that S3's geometric thinking process has developed to the level of formal deduction, according to Van Hiele's theory. At the stage of understanding the problem, S3 reads the problem thoroughly, identifies known information, such as the distances from the ship to the port and from Island 1 to Island 2, and writes this information down as the basis for solving the problem. In the devise-a-plan stage, S3 represents the problem situation with a sketch of the ship's trajectory that forms a right triangle, providing information about each side of the image. At the visualization level, S3 can recognize the shape of a right triangle based on the ship's trajectory, which is perpendicular to each other. Furthermore, at the level of analysis, S3 can mention the properties of triangles, such as having three sides and one right angle, and

Corresponding author: Venissa Dian Mawarsari (venissa@unimus.ac.id)

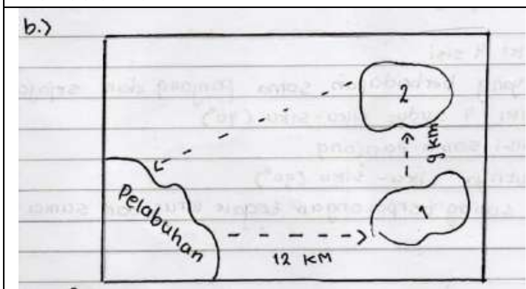
e-issn: 2746-1467 Journal of Education and Teaching (JET) Volume 7 No. 3, 2026
p-issn: 2747-2868 DOI: 10.51454/jet.v7i3.944

begin to connect these concepts to the Pythagorean Theorem to determine the lengths of the oblique sides of a triangle. This shows that S3 has been able to connect visual representations to geometric concepts relevant in problem-solving and planning.

In carrying out the plan stage, S3 shows the ability of informal deduction by using the Pythagorean Theorem precisely to determine the distance from the port to Island 2 as the oblique side of the right triangle. S3 then proceeds to complete the completion process at the formal deduction stage by performing logical reasoning and systematic calculations to determine the overall trajectory of the ship using the concept of triangular circumference. The results of the calculations are correct, and S3 can explain the steps to completion in a sequential manner. At the rigor stage, S3 can give the reason that the oblique side is the longest side of a right triangle, but the explanation given is still general. It does not show formal proof of the relationship between geometric concepts. Meanwhile, at the looking back stage, S3 re-examined the calculation results and tried to reconfirm the accuracy of the settlement steps used. The results of the interviews also show that S3 is aware of potential errors in certain calculations and tries to correct them independently. These findings suggest that the S3's geometric thinking ability has developed to the level of formal deduction, with a fairly good level of reflection. However, the ability to provide rigorous mathematical justification still needs further strengthening.

Level 4 Student Geometric Ability Analysis: Rigor

Table 7. Level 4 Student Geometric Ability Analysis: Rigor

Answer	Analysis of Geometric Thinking Processes
Understanding the Problem	
a.) Diketahui : - Jarak dari pelabuhan ke pulau 1 = 12 km arah timur. - Jarak dari Pulau 1 ke pulau 2 = 9 km arah utara. - setelah dari Pulau 2 kembali ke Pelabuhan	Able to identify information by writing down information that is known to be important in the question.
Devise a Plan	
	**Visualization:** able to draw a sketch of the ship's sailing route according to the information and provide captions on the drawing.
c.) Segitiga siku-siku .	Mention the shape of the ship's trajectory correctly, namely a right triangle.
D) - Memiliki tiga sisi - Satu sudut 90° - Dua sisi tegak lurus - Panjang sisi dapat dihitung dengan Teorema Pythagoras.	**Analysis:** can correctly mention the properties of a right triangle, including the characteristic of a right triangle, which is one of the 90^0 spoons. As well as being able to plan a strategy precisely to determine the length of the entire trajectory using the

Corresponding author: Venissa Dian Mawarsari (venissa@unimus.ac.id)

	circumference of the triangle, and find the oblique side using the Pythagoras formula.
Carry out the Plan	
E) Jarak Pelabuhan ke Pulau 2 = (Rumus Pythagoras) $c^2 = a^2 + b^2$ $= 12^2 + 9^2$ $= 144 + 81$ $= \sqrt{225}$ = 15 KM Jadi, Jarak dari pelabuhan ke pulau 2 adalah 15 KM.	**Formal deduction: determining the length of the passage from the Port to Island 2 using the Pythagoras** formula, which is then used to determine the entire length of the ship's passage (circumference of the triangle). As well as making calculations logically and correctly.
F) Jarak Total = 12 KM + 9 KM + 15 KM = 36 KM	**Formal deduction:** Write down the way to calculate the circumference of a right triangle correctly.
G) ya, karena Pelabuhan ke pulau 2 15 KM, Sedangkan pelabuha ke pulau, 12 KM.	**Rigor:** carry out logical reasoning by proving the problem from the statement, accompanied by the correct reason.

The results of the geometric thinking ability test in Table 7 are an analysis in Subject 4 (S4). Furthermore, an in-depth interview revealed that S4's geometric thinking process has developed to a level of rigor according to Van Hiele's theory. At the stage of understanding the problem, S4 can read it thoroughly, identify key information, and record known data as the basis for solving it. At the devise-a-plan stage, S4 can represent the ship's trajectory with a complete sketch, including information on each side and point of the trajectory. At the visualization level, S4 can recognize the shape of the trajectory as a right triangle based on the position of the trajectory that is perpendicular to each other. Furthermore, at the level of analysis, S4 can mention the properties of a right triangle in full, such as having one angle of 90°, two sides that are perpendicular to each other, and an oblique side that can be determined using the Pythagorean Theorem. S4 can also precisely plan the completion strategy by determining the oblique using the Pythagorean Theorem and calculating the ship's entire trajectory using the concept of the triangular circumference. This shows that S4 has been able to integrate visual representations, conceptual understanding, and relationships among geometric properties systematically.

In carrying out the plan stage, S4 demonstrates the ability of informal deduction by using the Pythagorean Theorem precisely to determine the length of the trajectory from the port to Island 2 as the oblique side of the right triangle. Furthermore, at the formal deduction stage, S4 can perform logical reasoning and systematic calculations to determine the ship's overall trajectory using the formula for the circumference of a right triangle, yielding the correct result. At the rigor stage, S4 can provide a precise mathematical justification by explaining that the oblique side is the longest side of a right triangle, using side-length comparisons and relationships among geometric concepts. In addition, at the looking-back stage, S4 re-checks each completed step and the calculation results to ensure the accuracy of the answers. The results of the interview showed that S4 found no errors after re-checking, indicating good reflective and evaluative skills. These findings show that the S4 geometric thinking process develops very well because it systematically integrates conceptual understanding, logical reasoning,

Corresponding author: Venissa Dian Mawarsari (venissa@unimus.ac.id)

mathematical proof, and reflection to reach the highest level of geometric thinking.

Relationship Between Van Hiele Levels and Polya's Problem-Solving Stages

The results of the analysis of students' geometric thinking processes show a close relationship between Van Hiele's geometric thinking levels and the problem-solving stages according to Polya. Students at each Van Hiele level demonstrate different abilities in understanding problems, developing solution plans, executing plans, and reviewing solutions. The higher the students' level of geometric thinking, the more systematic and complete the problem-solving process will be. The following is the relationship between Van Hiele's geometric thinking ability and the stages of Pólya's problem-solving, based on the thinking processes shown by subjects S0–S4.

Tabel 8. Relationship Between Van Hiele Levels and Polya's Problem-Solving Stages

Subject	Polya Problem Solving Stage			
	Understanding the Problem	Devise a Plan	Carry out the Plan	Looking Back
S0	Able to read questions and mention known information.	Able to draw a trajectory by mentioning the shape of the trajectory in the form of a triangle, but the mistake of recognizing the shape of the drawing and the mistake of identifying geometric properties make it impossible for students to design a solution.	Solving by adding the length of the sides arithmetically without using geometric relationships makes students unable to do reasoning and proofing.	Unable to conclude and not reviewing answers. Not aware of possible errors, so they do not make improvements.
S1	Able to write, write, write down information that is known and asked correctly.	Able to draw the trajectory of the ship according to the information correctly and correctly identify the properties of the right triangle, so it is used in planning the use of formulas.	Errors in the application of formulas (not being able to associate right triangles with Pythagoras) prevent them from completing logical calculations and proofs correctly.	Do not evaluate the correctness of the answers written.
S2	Able to understand information and questions well.	Able to draw the trajectory of the ship and correctly identify the properties of the right triangle, and to be able to relate concepts to design solutions.	Able to perform informal analysis using the concept of Pythagoras in determining the length of the oblique side, yet making mistakes in calculating the	Re-examining some of the answers, but not yet systematically. Not aware of any miscalculation.

Corresponding author: Venissa Dian Mawarsari (venissa@unimus.ac.id)

			circumference of the triangle. So that it cannot prove the statement.	
S3	Understand the problem thoroughly and write down information systematically.	Draw the trajectory and correctly identify the properties of the right triangle so that it can choose the right completion strategy.	Carry out calculations correctly and systematically, but cannot explain/justify the proof properly.	Evaluate the answers and review each step of the solution, but the justification is still incomplete.
S4	Understand the problem in depth, identify important information.	Draw the trajectory and correctly identify the properties of the right triangle so that it can strategize the completion efficiently.	Carry out calculations precisely, completely, and show a logical explanation of each step to prove.	Review thoroughly, check the consistency of the answers, and provide logical and correct justifications.

Table 8 shows the relationship between Polya's stages of problem solving and Van Hiele's development of geometric thinking skills in solving geometric problems. In general, the problem-solving process starts with the understanding-the-problem stage. At this stage, students read, recognize information, and understand the problem situation. After understanding the problem, students enter the devise stage of a plan. At this stage, geometric thinking skills begin to emerge: the visualization level (students recognize the shape of the building) and the Analysis level (students begin to identify the properties of the geometric building). Next, students enter the carry-out-the-plan stage. At this stage, the ability to think geometrically develops to a higher level, namely: the informal deduction level (students begin to relate geometric properties and use concepts such as the Pythagorean Theorem), the formal deduction level (students can formulate logical and systematic solution steps), and the Rigor level (students can provide mathematical reasons and justifications formally). The last stage is looking back, where students re-examine the answers, the completion steps, and the correctness of the results.

Discussion

Polya–Van Hiele Integration Hierarchical Model in Geometric Thinking Processes

The findings of this study indicate that students' geometric thinking processes are not static but develop hierarchically, progressing from visualization to rigor, and appear real, especially during the stages of planning and implementing problem-solving. Thus, this study successfully identifies and describes students' geometric thinking processes within the framework of Polya–Van Hiele integration, showing that geometry problem solving not only involves procedural steps but also reflects the Development of students' geometric reasoning quality. These results provide a solid basis for the Development of a geometry learning model that is more oriented to students' thinking processes, gradually and

Corresponding author: Venissa Dian Mawarsari (venissa@unimus.ac.id)

systematically. The following chart shows students' geometric thinking processes within the framework of Polya–Van Hiele integration.

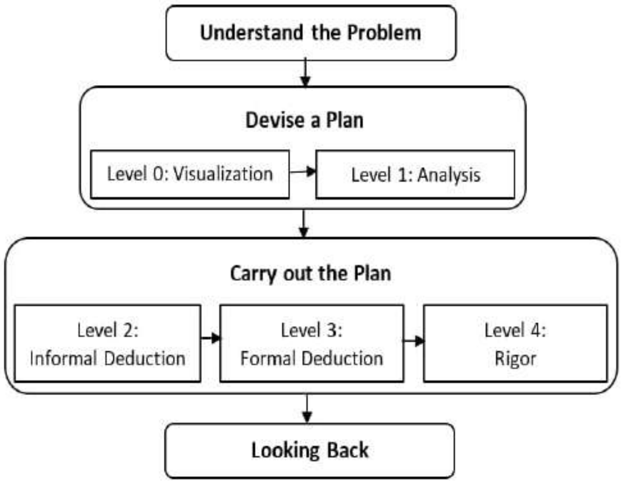


Figure 2. Polya–Van Hiele Integration Hierarchical Model in Students' Geometric Thinking Processes

The indicators of students' geometric thinking process at each stage are presented in Table 9.

Table 9. Geometric Process Indicators On the Polya–Van Hiele Integration Hierarchy Model

Polya Stage	Level Van Hiele	Indicators of the Geometric Thinking Process
Understand the Problem		1) Reading and understanding geometry problems. 2) Identify known information. 3) Identify questions.
Devise a Plan	Level 0: Visualization	1) Visually recognize geometric shapes. 2) Drawing or representing geometric shapes according to the problem. 3) Mention the type of build based on the image.
	Level 1: Analysis	1) Identifying the properties of geometric shapes. 2) Mention the relationship between elements (sides, angles). 3) Using the wake-up trait to determine the solution strategy.
Carry out the Plan	Level 2: Informal Deduction	1) Connecting relevant geometric concepts. 2) Using formulas (e.g., Pythagorean Theorem). 3) Explain the relationship between building elements in simple terms.
	Level 3: Formal Deduction	1) Systematically compile settlement steps. 2) Using logical reasoning in solutions. 3) Solve problems in an appropriate and structured manner.
	Level 4: Rigor	1) Provide a correct mathematical reason or justification. 2) Compare and generalize geometric concepts. 3) Demonstrate an in-depth understanding of the relationships between concepts.
Looking Back		1) Re-check the completion steps.

Corresponding author: Venissa Dian Mawarsari (venissa@unimus.ac.id)

		2) Evaluating the correctness of the results. 3) Identify and fix errors. 4) Reflecting on the thought process carried out.

Table 9 shows that the indicators of students' geometric thinking processes not only follow the stages of Polya's problem-solving but also reflect the Development of the Van Hiele geometric thinking levels, which emerge gradually at each stage of problem-solving. At the stage of understanding the problem, students tend to identify information and represent the problem as the basis for the solution. This shows that understanding problems and initial representations are important foundations for building mathematical problem-solving processes, including in geometry.

This shows that, at the stage of understanding the problem, students generally identify the known information by rewriting the important information in the question. This stage shows the initial process of geometric thinking before students enter more formal geometric reasoning. According to Polya, understanding the problem is the main basis for successful problem-solving. In contrast, in Van Hiele's theory, this stage is associated with the emergence of visualization skills through the recognition of geometric shapes and their representations. Students at the visualization level tend to recognize the building only by its appearance, while students at a higher level begin to understand the relationships among geometric elements more systematically.

In the devise-a-plan stage, students begin to demonstrate visualization and analysis skills by drawing geometric representations and identifying the building's properties. Students at the visualization level recognize right-angled triangles based on their appearance. In contrast, students at the analysis level can identify properties of triangles, such as right angles, perpendicular sides, and oblique sides. This finding is in line with Van Hiele's theory, which states that the development of geometric thinking starts from visual recognition of shapes to conceptual understanding of the properties of buildings. In addition, students' completion strategies become more structured as the level of geometric thinking increases.

At the carry-out-the-plan stage, students differ in their geometric reasoning skills. Students at lower levels tend to use intuitive logic and simple arithmetic operations without connecting relevant geometric concepts. On the other hand, students at higher levels can use the Pythagorean Theorem appropriately and formulate solution steps systematically. At the level of formal deduction, students begin to construct logical reasoning and devise structured settlement procedures. Meanwhile, students at the rigor level have been able to provide mathematical justifications and generalize concepts formally. These findings show that the quality of problem-solving implementation is greatly influenced by the development of students' geometric thinking skills.

Looking back, reflective ability is an important indicator in problem-solving. Students with low geometric thinking skills generally do not double-check the answers and have difficulty in identifying errors systematically. On the other hand, high-achieving students can re-examine the steps for completion, evaluate the results, and reflect on the strategies used. This shows that the ability to reflect

Corresponding author: Venissa Dian Mawarsari (venissa@unimus.ac.id)

develops as the level of geometric thinking increases and becomes an important part of high-level geometric reasoning.

Comparison with Previous Research

The results of this study indicate that during the planning stage of the solution, students began to use visualization and analytical skills, such as recognizing building shapes and identifying their properties. These results are consistent with Van Hiele's theory, which states that the development of geometric thinking begins with recognizing shapes and then understanding their properties (Siagian et al., 2020). These findings are also supported by other researchers, who show that students' ability to relate visual representations to geometric properties is an important indicator of the transition from visualization to analysis (Sunardi et al., 2019; Uwurukundo et al., 2022). However, this study also found that not all students consistently use these traits to determine completion strategies, suggesting that the Development of geometric thinking is gradual and not always linear (Fitriyani et al., 2018; Celik & Yilmaz, 2022).

At the stage of implementing the plan, the study's results showed that the students' geometric thinking process developed to include the levels of informal deduction, formal deduction, and rigor. Students begin to use concepts such as the Pythagorean Theorem, connect various geometric concepts, and systematically formulate solution steps. At this stage, students with higher geometric thinking abilities are not only able to solve problems procedurally but also exhibit thinking characteristics at a level of rigor that enables them to provide correct mathematical reasons or justifications when proving statements. However, some students still make errors in applying concepts and performing calculations, which shows that the ability to reason geometrically develops gradually. These findings are in line with other studies that state that high school students' mathematical reasoning skills develop through the process of connecting concepts and constructing logical arguments, but often still face constraints on consistency in concept use and procedural rigor (Basir et al., 2022; Patac et al., 2022; Baidowi et al., 2023). In addition, this study shows that students with higher abilities integrate concepts and procedures more effectively than those with lower abilities, indicating a relationship between geometric thinking levels and problem-solving quality.

At the re-examination stage, some students demonstrate their reflection skills by revisiting the steps and the results obtained. This activity helps strengthen understanding and ensure the solution's correctness, although not all students do it optimally. This shows that reflective skills still need to be developed in the learning process. This emphasizes the importance of formal justification and reasoning skills in high-level geometric thinking, although in this study, these skills have emerged more dominant at the stage of implementation completion (Atasoy, 2019; Suraya et al., 2019; Bonyah & Larbi, 2021).

Compared with previous studies, most examined geometric thinking abilities based on Van Hiele's theory or problem-solving based on Polya's steps separately. This research makes a distinct contribution by integrating the two approaches in

Corresponding author: Venissa Dian Mawarsari (venissa@unimus.ac.id)

a single conceptual model. In contrast to studies that describe only the student's Van Hiele level, this study shows how the level develops in each stage of problem-solving. Thus, this study expands on the previous study by presenting the Polya–Van Hiele Integration Hierarchy Model, which explains the relationship between the problem-solving process and the Development of students' geometric thinking more comprehensively.

Overall, the results of this study confirm that the quality of geometry problem-solving is strongly influenced by students' level of geometric thinking development. The higher the level of geometric thinking, the more systematic, logical, and reflective the problem-solving process will be. Therefore, geometry learning needs to be designed to encourage the gradual development of students' thinking from visualization to formal reasoning, and to provide space for students to reflect on their thinking process. The model produced in this study can be a reference in understanding and developing students' geometric thinking processes in a more structured manner.

Implications for Learning Practice

The results of this study show that geometry learning should go beyond simply emphasizing problem-solving procedures. It needs to support the gradual development of students' geometric thinking skills. Teachers should design learning activities that help students progress through visualization, analysis, informal deduction, formal deduction, and rigor. Geometry lessons must also highlight relationships among visual representations, building properties, and formal mathematical reasoning in a systematic way. Integrating the Polya stage and Van Hiele level offers a practical framework for planning geometry learning and assessment. Teachers can identify students' levels of geometric thinking across different stages of problem-solving and provide appropriate scaffolding. Reflection activities, such as double-checking answers, explaining mathematical reasons, and evaluating errors, should be strengthened to develop students' high-level geometric and metacognitive abilities.

CONCLUSION

Based on the research and discussion, it can be concluded that students' geometric thinking in solving geometric problems develops gradually and is integrated into the Polya problem-solving stages. At the stage of understanding the problem, students identify information and initial representations as the basis for the solution. At the stage of planning the completion, the geometric thinking process begins to emerge through visualization and analysis. In contrast, at the stage of implementing the plan, it develops to the level of informal, formal, and rigorous deduction. The re-examination stage plays a role in strengthening reflection and evaluation of the settlement process.

This study led to the Polya–Van Hiele Integrated Hierarchical Model, which demonstrates that the development of geometric thinking is embedded in mathematical problem-solving activities. The findings imply that geometry learning should focus not only on final answers but also on the gradual

Corresponding author: Venissa Dian Mawarsari (venissa@unimus.ac.id)

development of students' thinking processes. Teachers are encouraged to design learning activities that integrate problem-solving stages with the development of geometric thinking, including opportunities for students to represent problems, analyze geometric properties, connect concepts, and evaluate solutions. The proposed model may also serve as a reference for developing instructional strategies and assessment instruments in geometry learning.

Several recommendations can be proposed based on this study. First, mathematics teachers are encouraged to integrate Polya's problem-solving stages with the development of students' geometric thinking in classroom instruction. Second, future studies should examine the applicability of the Polya–Van Hiele Integrated Hierarchical Model in different educational levels and geometry topics. Third, further research may develop the model into technology-based learning media or instructional tools to support geometry learning more effectively.

ACKNOWLEDGMENT

The authors would like to express their sincere gratitude to the Lembaga Penelitian dan Pengabdian kepada Masyarakat (LPPM), Universitas Muhammadiyah Semarang, for funding this study through the UNIMUS Internal Grant under Grant Agreement No. 003/UNIMUS.L/PG/PDP/PJ.INT/20205. The authors also extend their appreciation to the Principal, Mathematics Teacher, and students of SMP Negeri 19 Semarang, Central Java, Indonesia, for their willingness to participate in this study. The authors are grateful to the Undergraduate Mathematics Education Study Program of Universitas Muhammadiyah Semarang for its support in the implementation of this research. Appreciation is also extended to the Doctoral Program in Mathematics Education at Universitas Negeri Semarang for its collaboration and support throughout the research process. The authors further acknowledge the contributions of all research team members: VDM, who conceived and conducted the study; MP, who carried out the literature review; EAP, who developed the research instruments; YLS, who validated the research framework; SM, who supervised the data analysis; and IJ, who monitored and oversaw the research process.

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Corresponding author: Venissa Dian Mawarsari (venissa@unimus.ac.id)